

1-1

Variables and Expressions

Common Core State Standards

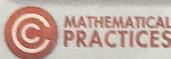
A-SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.

MP 1, MP 3, MP 4, MP 7

Objective To write algebraic expressions



Can the number of states in the United States vary?



Getting Ready!

Consider the **population of Florida**, the **area of Colorado**, and the **flight time from Philadelphia to San Francisco**. Which of these has a value that varies? Explain.



A mathematical **quantity** is anything that can be measured or counted. Some quantities remain constant. Others change, or vary, and are called **variable quantities**.

Essential Understanding Algebra uses symbols to represent quantities that are unknown or that vary. You can represent mathematical phrases and real-world relationships using symbols and operations.

A **variable** is a symbol, usually a letter, that represents the value(s) of a variable quantity. An **algebraic expression** is a mathematical phrase that includes one or more variables. A **numerical expression** is a mathematical phrase involving numbers and operation symbols, but no variables.



Lesson Vocabulary

- quantity
- variable
- algebraic expression
- numerical expression

Plan

How can a diagram help you write an algebraic expression?

Models like the ones shown can help you to visualize the relationships described by the word phrases.



Problem 1 Writing Expressions With Addition and Subtraction

What is an algebraic expression for the word phrase?

Word Phrase	Model	Expression
A 32 more than a number n $n + 32$		$n + 32$
B 58 less a number n $58 - n$ 58 less than a number n $n - 58$		$58 - n$



Got It? 1. What is an algebraic expression for 18 more than a number n ?

Think

Is there more than one way to write an algebraic expression with multiplication? Yes. Multiplication can be represented using a dot or parentheses in addition to an \times .

Problem 2 Writing Expressions With Multiplication and Division

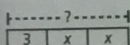
What is an algebraic expression for the word phrase?

Word Phrase	Model	Expression
A 8 times a number n $8 \times n$ $8 \cdot n$ $8(n)$ $8 \cdot n$ $8n$		$8 \times n$, $8 \cdot n$, $8n$
B the quotient of a number n and 5 $n \div 5$ $\frac{n}{5}$ $n/5$		$n \div 5$, $\frac{n}{5}$

- Got It? 2. What is an algebraic expression for each word phrase in parts (a) and (b)?
- 6 times a number n
 - the quotient of 18 and a number n
- c. **Reasoning** Do the phrases 6 less a number y and 6 less than a number y mean the same thing? Explain.

Plan

How can I represent the phrases visually? Draw a diagram. You can represent the phrase in Problem 2, part (A), as shown below.



Problem 3 Writing Expressions With Two Operations

What is an algebraic expression for the word phrase?

Word Phrase	Expression
A 3 more than twice a number x $2x + 3$	$3 + (2x)$
B 9 less than the quotient of 6 and a number x $\frac{6}{x} - 9$	$\frac{6}{x} - 9$
C the product of 4 and the sum of a number x and 7 $4(x + 7)$	$4(x + 7)$

- Got It? 3. What is an algebraic expression for each word phrase?
- 8 less than the product of a number x and 4
 - twice the sum of a number x and 8
 - the quotient of 5 and the sum of 12 and a number x

In Problems 1, 2, and 3, you were given word phrases and wrote algebraic expressions. You can also translate algebraic expressions into word phrases.

Think

Is there only one way to write the expression in words? No. The operation performed on 3 and x can be described by different words like "multiply," "times," and "product."

Problem 4 Using Words for an Expression

What word phrase can you use to represent the algebraic expression $3x$?

Expression	Words
$3x$ $3 \cdot x$	A number and a variable side by side indicate a product.

Words three times a number x or the product of 3 and a number x

- Got It? 4. What word phrase can you use to represent the algebraic expression?
- $x + 8.1$
 - $10x + 9$
 - $\frac{n}{3}$
 - $5x - 1$

You can use words or an algebraic expression to write a mathematical rule that describes a real-life pattern.

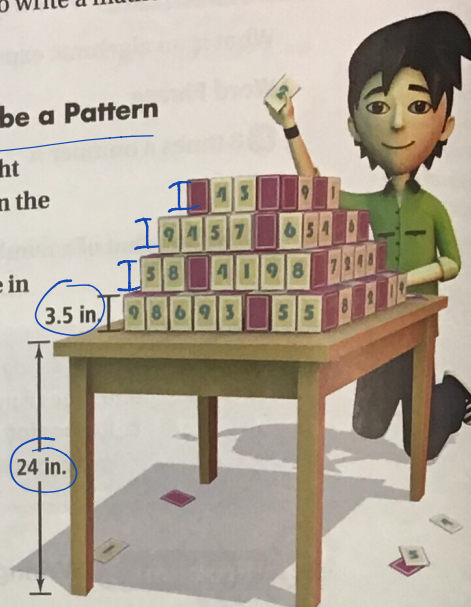
Problem 5 Writing a Rule to Describe a Pattern

Hobbies The table below shows how the height above the floor of a house of cards depends on the number of levels.

A What is a rule for the height? Give the rule in words and as an algebraic expression.

House of Cards

Number of Levels	Height (in.)
2	$(3.5 \cdot 2) + 24$
3	$(3.5 \cdot 3) + 24$
4	$(3.5 \cdot 4) + 24$
n	? $3.5n + 24$



Know

Numerical expressions for the height given several different numbers of levels

Need

A rule for finding the height given a house with n levels

Plan

Look for a pattern in the table. Describe the pattern in words. Then use the words to write an algebraic expression.

Rule in Words

Multiply the number of levels by 3.5 and add 24.

Rule as an Algebraic Expression

The variable n represents the number of levels in the house of cards.

$$3.5n + 24$$

This expression lets you find the height for n levels.

B A group of students built another house of cards that had 10 levels. Each card was 4 inches tall, and the height from the floor to the top of the house of cards was 70 inches. How tall would the house of cards be if they built an 11th level?

Since each card was 4 inches tall, adding 1 more level would increase the total height of the house of cards by 4 inches.

The house of cards would be $70 + 4$, or 74 inches tall if the 11th level were added.

C Another group of students built a third house of cards with n levels. Each card was 5 inches tall, and the height from the floor to the top of the house of cards was $34 + 5n$ inches. How tall would the house of cards be if the group added 1 more level of cards?

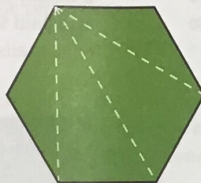
height of table

Since each card was 5 inches tall, adding 1 more level would increase the total height of the house of cards by 5 inches.

The house of cards would be $34 + 5n + 5$ in. tall if the next level were added.



- Got It?** 5. Suppose you draw a segment from any one vertex of a regular polygon to the other vertices. A sample for a regular hexagon is shown below. Use the table to find a pattern. What is a rule for the number of nonoverlapping triangles formed? Give the rule in words and as an algebraic expression.



Triangles in Polygons

Number of Sides of Polygon	Number of Triangles
4	$4 - 2$
5	$5 - 2$
6	$6 - 2$
n	



Lesson Check

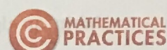
Do you know HOW?

- Is each expression *algebraic* or *numerical*?
 a. $7 \div 2$ b. $4m + 6$ c. $2(5 - 4)$
- What is an algebraic expression for each phrase?
 a. the product of 9 and a number t
 b. the difference of a number x and $\frac{1}{2}$
 c. the sum of a number m and 7.1
 d. the quotient of 207 and a number n

Use words to describe each algebraic expression.

- $6c$
- $x - 1$
- $\frac{t}{2}$
- $3t - 4$

Do you UNDERSTAND?



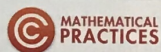
- Vocabulary** Explain the difference between numerical expressions and algebraic expressions.
- Reasoning** Use the table to decide whether $49n + 0.75$ or $49 + 0.75n$ represents the total cost to rent a truck that you drive n miles.

Truck Rental Fees

Number of Miles	Cost
1	$\$49 + (\$.75 \times 1)$
2	$\$49 + (\$.75 \times 2)$
3	$\$49 + (\$.75 \times 3)$
n	



Practice and Problem-Solving Exercises



Practice

Write an algebraic expression for each word phrase.

- 4 more than p
- the quotient of n and 8
- a number t divided by 82
- 6.7 more than the product of 5 and n

Write a word phrase for each algebraic expression.

- $q + 5$
- $\frac{y}{5}$
- $9n + 1$
- $\frac{z}{8} - 9$

- y minus 12
- the product of 15 and c
- the sum of 13 and twice a number h
- 9.85 less than the product of 37 and t

See Problems 1–3.

See Problem 4.

- $12x$
- $49 + m$
- $15 - \frac{1.5}{d}$
- $2(5 - n)$



Vocabulary

● Review

What mathematical *operation* is shown in each equation? Write *addition*, *subtraction*, *multiplication*, or *division*.

1. $6 \cdot 2 = 12$

multiplication

2. $14 - 4 = 10$

subtraction

3. $27 \div 3 = 9$

division

4. $13 + 7 = 20$

addition

● Vocabulary Builder

variable (noun) VEHR ee uh bul

Related Words: vary (verb), varied (adjective), various (adjective)

Definition: A **variable** is a symbol, usually a letter, that represents one or more values of a quantity that changes.

Main Idea: The value given to a **variable** can change or vary. A quantity that changes, or varies, is called a *variable quantity*.

Example: The letter y is the **variable** in the algebraic expression $4 + y$. You can replace y with different numbers to find values for the expression.

a , x , and m are often used as **variables**.
 100 , $\frac{1}{a}$, and $3m$ are **not variables**.

$$\begin{aligned} 4 + 1 &= 5 \\ 4 + 10 &= 14 \end{aligned}$$

● Use Your Vocabulary

5. Circle the *variable(s)* in each algebraic expression.

$8 + 4x$

$y + 12$

$9z + y$

$\frac{8}{w} + 4w$

An **algebraic expression** is a mathematical phrase that includes one or more variables. A **numerical expression** is a mathematical phrase involving numbers and operation symbols, but no variables.

6. Write N next to each *numerical expression*. Write A next to each *algebraic expression*.

A $6x$

A $\frac{5}{0} - 4$

N $11 + 5$
16

A $30 + 14x$



Problem 1 Writing Expressions With Addition and Subtraction

Got It? What is an algebraic expression for 18 more than a number n ?

7. Complete the table with *add* or *subtract*.

Phrase	Math Operation
more than a number	<u>add</u>
less a number	<u>subtract</u>
sum of two numbers	<u>add</u>
fewer than a number	<u>subtract</u>

Add
more than
sum
plus
increased by
added to
total of
in all
all together

Subtract
less
less than
fewer than
decreased by
difference
minus

8. Circle the expression you could use to find 18 more than 6.

6×18 $6 + 18$ $18 - 6$ $18 + 18 + 18 + 18 + 18 + 18$

9. Now write an algebraic expression for 18 more than a number n .

$n + 18$



Problem 2 Writing Expressions With Multiplication and Division

Got It? What is an algebraic expression for the following word phrase?

6 times a number n

10. Complete each sentence with *add*, *subtract*, *multiply*, or *divide*. One word is used more than once.

The phrase "8 less than a number" tells you to ? 8.

subtract

The phrase "the product of a number x and 4" tells you to ? x and 4.

multiply

The phrase "the quotient of 6 and a number" tells you to ? 6 by x .

divide

The phrase " n times 12" tells you to ? n and 12.

multiply

The phrase "the sum of a number n and 59" tells you to ? n and 59.

add

11. Now write an algebraic expression for 6 times a number n .

$6 \cdot n$
 $6(n)$
 $6n$

multiply
times
product of
doubled (times 2)
tripled (times 3)
of
per

division
quotient
divided by
over
per



Problem 3 Writing Expressions With Two Operations

Got It? What is an algebraic expression for the following word phrase?

8 less than the product of a number x and 4

12. Write an algebraic expression for the product of a number x and 4.

$$4x$$

13. Underline the correct phrase to complete the sentence.

The phrase "8 less than a certain number" tells you to

subtract 8 from a number / subtract a number from 8.

14. Cross out the expressions that do NOT represent the word phrase "8 less than the product of a number x and 4."

$$4x - 8$$

$$4x + 8$$

$$x - 8$$

$$8x - 4$$



Problem 4 Using Words for an Expression

Got It? What word phrase can you use to represent each algebraic expression?

$$x + 8.1$$

$$10x + 9$$

$$\frac{n}{3}$$

$$5x - 1$$

15. Complete the word phrase for each expression.

the ? of a number x and 8.1

Sum

the ? of 10 ? a number x and 9

Sum

times

the quotient of ? and 3

a number n

1 ? the product of ?

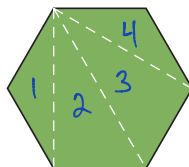
less than

5 and x



Problem 5 Writing a Rule to Describe a Pattern

Got It? Suppose you draw a segment from any one vertex of a regular polygon to the other vertices. A sample for a regular hexagon is shown at the right. Use the table to find a pattern. What is a rule for the number of nonoverlapping triangles formed? Give the rule in words and as an algebraic expression.



Triangles in Polygons

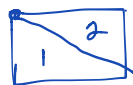
Number of Sides of Polygon	Number of Triangles
4	4 - 2
5	5 - 2
6	6 - 2
n	$n - 2$

16. Use the table. Find the number of nonoverlapping triangles in each figure.

a polygon with 4 sides



a polygon with 5 sides



17. Underline the correct word or words to complete the sentence.

The value of the expression in the table for a 6-sided figure is / is not the same as the number of triangles in the drawing of the hexagon.

18. Give a rule in words to find the number of nonoverlapping triangles in a polygon.

19. Write an algebraic expression for the number of nonoverlapping triangles in a polygon that has n sides.



Lesson Check • Do you UNDERSTAND?

Reasoning Use the table to decide whether $49n + 0.75$ or $49 + 0.75n$ represents the total cost to rent a truck that you drive n miles.

Truck Rental Fees

Number of Miles	Cost
1	$\$49 + (\$.75 \times 1)$
2	$\$49 + (\$.75 \times 2)$
3	$\$49 + (\$.75 \times 3)$
n	

20. Write a rule in words for the pattern shown in the table.

The sum of \$49 and \$.75 times the number of miles.
 $49 + 0.75n$

21. Now write an algebraic expression to represent the total cost of renting a truck.



Math Success

Check off the vocabulary words that you understand.



variable



algebraic expression



numerical expression

Rate how well you can write algebraic expressions.

